Course 5

Wednesday, 24 September 2025 09:54

Course 5 - Exercise...

EPFL

Génie Electrique et Electronique
Master program
Prof. Elison Matioli

EE-557
Semiconductor devices I

Doping and carrier concentration

Outline of the lecture

Exercise list 1 - corrections

References:

- J. A. del Alamo, course materials for 6.720J Integrated Microelectronic Devices, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu/)

Exercise list 1

EPFL

1. Apply the graphic method to determine the Fermi level for a boron-doped silicon sample with an impurity concentration of 10^{15} cm^{-3} at 300 K (note $n_i = 9.65 \times 10^9$ cm^{-3}).

P-type

$N_A = 10^{15}\ cm^{-3}$

Boron :

$E_A - E_V = 0.044 eV$

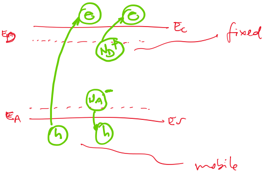
I $\boxed{N_A^- = \dfrac{N_A}{1 + 4\exp\left(\dfrac{E_A - E_F}{kT}\right)}}$

Charge neutrality :

$\cancel{n} + \underline{\underline{N_A^-}} = p + \cancel{N_D^+}$

$\boxed{p = N_A^-}$

II $\boxed{p = N_V \cdot \exp\left(-\dfrac{E_F - E_V}{kT}\right)}$



Charge neutrality

$\boxed{n + N_A^- = p + N_D^+}$

$$N_A^- = \frac{N_A}{1 + 4\exp\left[\dfrac{(\overbrace{E_A - E_V}^{0.044}) - (E_F - E_V)}{kT}\right]}$$

$$\underset{\substack{\| \\ 3.1 \times 10^{19}\ cm^{-3}}}{N_V} \cdot \exp\left(-\frac{E_F - E_V}{kT}\right) = \frac{N_A \overset{}{=} 10^{15}}{1 + 4\exp\left[\dfrac{0.044 - (E_F - E_V)}{kT}\right]}$$

if n-type material

$N_A \simeq 0$

$n = p + N_D^+ \simeq N_D^+$

$\boxed{n = N_D^+}$

electrons ionized donors

if intrinsic
$n = p = 10^{10}\ cm^{-3}$

$N_D = 10^{17 - 18 - 19}\ cm^{-3}$

if p-type material (if $N_A^- \gg n_i$)

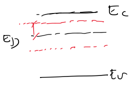
$\boxed{p = N_A^-}$

to calculate N_A^- or N_D^+ :

$$N_D^+ = \frac{N_D}{1 + g_D \cdot \exp\left(-\frac{E_D - E_F}{kT}\right)} = \boxed{\frac{N_D}{1 + 2\exp\left(-\frac{E_D - E_F}{kT}\right)}} \propto \exp\left(-\frac{E_F - E_D}{kT}\right)$$

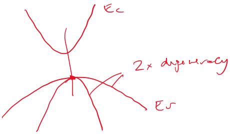
// cond. band

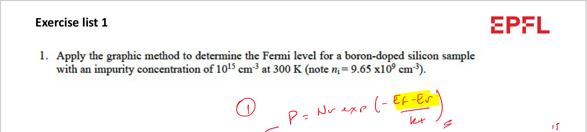
$g_D = 1 \times \text{spin} = 2$



$$N_A^- = \frac{N_A}{1 + g_A \cdot \exp\left(-\frac{E_F - E_A}{kT}\right)} = \boxed{\frac{N_A}{1 + 4\exp\left(-\frac{E_F - E_A}{kT}\right)}}$$

$g_A = \underline{2} \times \underset{\text{spin}}{2} = 4$



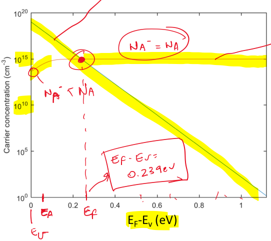


Exercise list 1

1. Apply the graphic method to determine the Fermi level for a boron-doped silicon sample with an impurity concentration of 10^{15} cm^{-3} at 300 K (note $n_i = 9.65 \times 10^9$ cm^{-3}).

① $P = N_V \exp\left(-\frac{E_F - E_V}{kT}\right)$

$N_A^- \simeq N_A$



$$N_A^- = \frac{N_A}{1+4\exp\left(-\frac{E_F-E_A}{kT}\right)}$$

②

$$= \frac{N_A}{1+4\exp\left(\frac{0.044-(E_F-E_V)}{kT}\right)}$$

$P = 10^{15}\ cm^{-3} = N_A$

Exercise list 1

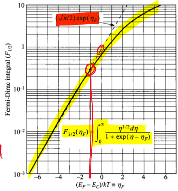
11. Determine the Fermi energy level and the maximum doping concentration at which the Boltzmann approximation is still valid. Assume T = 300 K, consider both n-type and p-type silicon.

$n = N_c \exp\left(-\frac{E_c - E_F}{kT}\right)$

for n-type Silicon:

$n = N_c \exp\left(-\frac{E_c - E_F}{kT}\right) \simeq N_D = N_D^+$

assuming all donors are ionized



$E_c - E_F \geq kT$

$n = 2.9\times10^{19} \exp\left(-\frac{kT}{kT}\right)$

$\boxed{N_D^+ = n = 1.05\times10^{19}\ cm^{-3}}$

for p-type:

$N_A = p = N_V \exp\left(-\frac{E_F - E_V}{kT}\right) \Rightarrow \boxed{N_A^- = 1.14\times10^{19}\ cm^{-3}}$

$7\ kT$

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3. Calculate the position of the Fermi level with respect to the bottom of the conduction band (E_C-E_F) for a silicon sample at 300 K, which is doped with 2×10^{10} cm^{-3} with fully ionized donors.

$N_D^+ = N_D$

if no doping:

$n = p$

$E_c - E_F = kT \ln(N_c/n)$

$E_F - E_V = kT \ln(N_V/p)$

(2)-(1) : $E_F - E_V - E_c + E_F = kT\left(\ln(N_c/n) - \ln(N_V/p)\right)$

$$\boxed{E_F = \underbrace{\frac{E_c + E_V}{2}}_{\text{mid-gap}} + \underbrace{\frac{kT}{2}\cdot\ln\left(\frac{N_V}{N_C}\right)}_{\approx 1\ meV}}$$

3.1, 2.9

$\frac{1.12}{2} = 0.56\ eV$

E_c
E_i
1 meV
E_V

$\boxed{E_F = 0.57\ eV = E_i}$

$n = N_c \cdot \exp\left(-\frac{E_c - E_F}{kT}\right) =$ if $E_F = E_i$, $n = n_i = 10^{10}\ cm^{-3}$

$$N_D^+ = N_D = \underline{2\times10^{10}}\ cm^{-3}$$

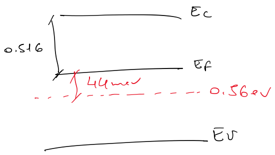
$$n + \cancel{N_A^-} = p + N_D^+ \Rightarrow \begin{cases} n = p + N_D^+ \\ \underline{n \cdot p = n_i^2} \end{cases}$$

$$n(n - N_D^-) = n_i^2$$

$$\begin{cases} n = \dfrac{N_D^+ \pm \sqrt{N_D^{+2} + 4n_i^2}}{2} = 2.41\times10^{10}\ cm^3 \\ p = \dfrac{n_i^2}{n} = 4.15\times10^{9}\ cm^{-3} \end{cases}$$

$$p = N_v \exp\left(-\frac{E_f - E_v}{kT}\right)$$

$$E_f - E_c = kT \ln\left(\overset{2.41\times10^{10}\ cm^{-3}}{n} \Big/ \underset{2.9\times10^{19}\ cm^{-3}}{N_c}\right) = \underline{-0.516\ eV}$$



4. Calculate the intrinsic carrier concentration in silicon at T = 300 K. The values of N_C and N_V for silicon at T = 300 K are 2.9×10^{19} cm^{-3} and 3.1×10^{19} cm^{-3}, respectively; the Si bandgap is 1.12 eV. Repeat the exercise for gallium nitride (GaN), assuming N_C and N_V at T = 300 K of 2.3×10^{18} cm^{-3} and 1.8×10^{19} cm^{-3}, respectively, and a bandgap of 3.4 eV. What do you conclude on the effect of the material on the intrinsic carrier concentration?

$$p \cdot n = n_i^2 = N_c \cdot N_v \cdot \exp\left(-\frac{E_g}{kT}\right)$$

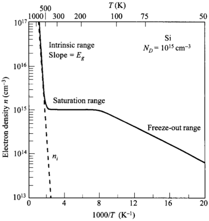
for Si at 300 K:

$$n_i = 1.22\times10^{10}\ cm^{-3} \simeq 10^{10}\ cm^{-3}$$

for GaN at 300K:

$$n_i = 2\times10^{-10}\ cm^{-3}$$

5. Estimate the temperature at which a p-Si wafer doped with $N_A = 10^{17} cm^{-3}$ becomes intrinsic (enters the intrinsic range).



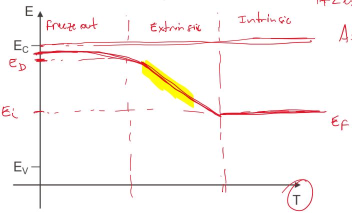
6. Consider an n-doped semiconductor (donor energy E_D). In the axis provided below, sketch the evolution of the location of the Fermi level with respect to the band edges as a function of temperature for the entire temperature range. For simplicity, assume that N_C, N_V are temperature independent. Explain your drawing in each of the temperature regimes. What if the semiconductor is p-doped? How does the curve changes with dopant concentration?

$$n = N_c \exp\left(-\frac{E_c - E_F}{kT}\right)$$

$$N_D^+ = \frac{N_D}{1+2\exp\left(-\frac{E_D - E_F}{kT}\right)}$$

N_D

Assumption
: E_g is constant



From course 3

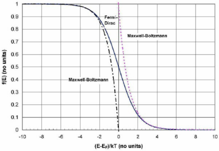
Electron statistics

Fermi-Dirac

$$f(E) = \frac{1}{1+\exp\frac{E-E_F}{kT}}$$

Maxwell-Boltzmann:

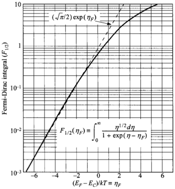
For $E - E_F \gg kT$:



$$f(E) \simeq \exp -\frac{E-E_F}{kT}$$

For $E-E_F \ll kT$:

$$f(E) \simeq 1-\exp\frac{E-E_F}{kT}$$



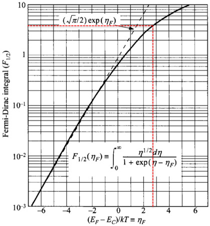
Energy Bands and Energy Gap

$$n = N_C\frac{2}{\sqrt{\pi}}F_{1/2}\left(\frac{E_F-E_C}{kT}\right)$$

where $N_C \equiv 2\left(\frac{2\pi m_{de}kT}{h^2}\right)^{3/2}$

$$F_{1/2}\left(\frac{E_F-E_C}{kT}\right) \equiv F_{1/2}(\eta_F) = \int_{E_C}^{\infty}\frac{[(E-E_C)/kT]^{1/2}}{1+\exp[(E-E_F)/kT]}\frac{dE}{kT}$$

$$= \int_0^\infty \frac{\eta^{1/2}}{1+\exp(\eta-\eta_F)}d\eta$$



Non-degenerate Semiconductors. By definition, in non-degenerate semiconductors, the doping concentrations are smaller than Nc and the **Fermi levels are more than several kT below Ec**, the Fermi-Dirac integral approaches:

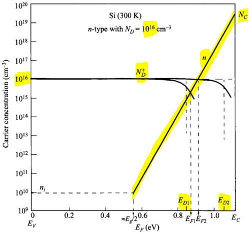
$$F_{1/2}\left(\frac{E_F-E_C}{kT}\right) = \frac{\sqrt{\pi}}{2}\exp\left(-\frac{E_C-E_F}{kT}\right)$$

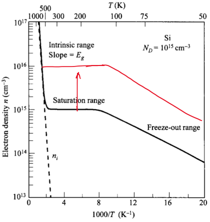
From course 4

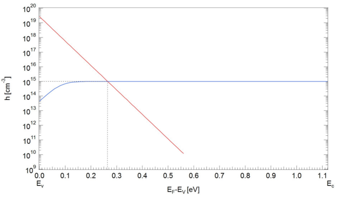
Graphical method:

Determine the Fermi energy level E_F and electron concentration n, when ionization is not complete.

Examples with two different values of impurity levels E_D are shown below:

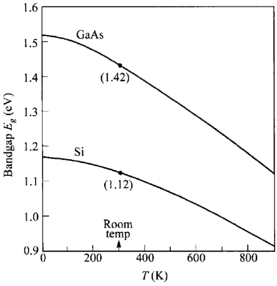






From course 2

Temperature dependence of the band gap



	$E_g(0)$ (eV)	α (eV/K)	β (K)
GaAs	1.519	5.4×10^{-4}	204
Si	1.169	4.9×10^{-4}	655

$$E_g(T) = E_g(0) - \frac{\alpha T^2}{T+\beta}$$

From course 4

(E_A)? to calculate N_A^- (ionized donor concentration)?

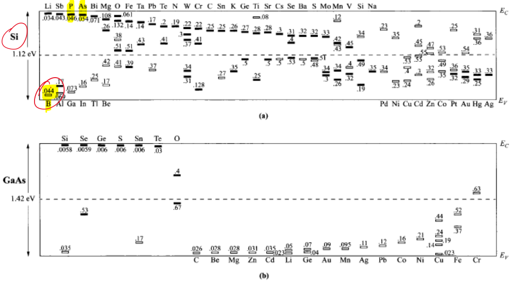


Fig. 10 Measured ionization energies for varies impurities in (a) Si and (b) GaAs. Levels below the gap center are measured from E_V. Levels above the gap center are measured from E_C. Solid bars represent donor levels and hollow boxes represent acceptor levels. (After Refs. 29, 31, 34, and 35.)